Larger Value and SI Measurement of the Improved Cryogenic Capacitor for the Electron-Counting Capacitance Standard

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Abstract—We report on several advances in the development of a cryogenic vacuum-gap capacitor C_{cryo}, for use with the electron-counting capacitance standard (ECCS). First, we have increased the value by about a factor of ten, to 10 pF; this will both make the ECCS more useful as a commercial standard and also allows a substantial reduction in the relative uncertainty of the calibration of C_{cryo}. Second, the capacitor's stability is excellent, with a relative drift less than 10^{-9}/hour. This stability is required for the third advance: we have succeeded in tuning the calculable capacitor, which allows us to make a measurement of SI units, without requiring us to fabricate the capacitor to have a precise value of C_{cryo}. We demonstrate such a measurement, with an uncertainty of about 4×10^{-8}.

Index Terms—Calculable capacitor, cryogenic vacuum-gap capacitor.

I. Introduction

THE electron-counting capacitance standard (ECCS) has been under development by workers at the National Institute of Standards and Technology (NIST) for several years [1]. It is based on the idea of forming a primary capacitance standard by counting about one hundred million electrons onto the plate of a cryogenic capacitor and measuring the voltage that develops [2]. Then, through the relation $Q = Ne = C_{\mathrm{cryo}}V$, we can determine the capacitance. Besides use as a "turnkey" commercial primary standard, other payoffs include measuring the fine-structure constant α and closing the quantum metrology triangle (by relating C_{cryo} back to the resistance quantum $R_{\mathrm{K\text{-}90}}$).

One key element in this experiment is the cryogenic capacitor C_{cryo}. We require that this capacitor have no frequency dependence between 0.1 Hz and 1 kHz, no voltage dependence between 0 and 10 V, and no leakage (minimum leakage resistance to ground 10^{21} Ω). By using a vacuum-gap capacitor [3], we have demonstrated achievement of the last constraint and the first two at about the 10^{-6} level (and are working to improve these) [3], [4].

In using the ECCS as a turnkey standard, its likely predominant role will be to provide a capacitor with a well-known value. This capacitor can be used to calibrate commercial capacitance bridges, which can measure a wide range of capacitance values. Thus, the value of C_{cryo} should be large enough to minimize the

effect of the commercial bridge noise floor and should also fit comfortably in the range of values one is likely to measure with such a commercial bridge.

Up until now, we have been working with cryogenic capacitors near 1 pF; this value does not fulfill either of the goals in the previous paragraph. Thus, the first major achievement reported herein is the demonstration of a stable cryogenic capacitor with a value close to 10 pF (about 3% higher).

In addition, we wish to provide a link between the ECCS and the SI unit of capacitance (necessary to measure α and close the metrology triangle, as well as to give us confidence in the usefulness of the ECCS as a primary standard). At least conceptually, this comparison need only be done once. At first glance, such a comparison would be easy to achieve, by using a high-accuracy capacitance bridge to compare C_{cryo} to an SI-derived standard. At NIST, we wish to make a comparison (directly and/or indirectly) between C_{cryo} and the calculable capacitor.

However, such a comparison is made more difficult by two facts: 1) the best capacitance bridges typically have dynamic ranges of only about 10^{-3}, thus restricting their use to capacitors with values very close to nominal and 2) it is not easy to construct a cryogenic capacitor with a nominal value (we note that one group has successfully done so for a 1-pF cryogenic capacitor [5]).

Thus, we have chosen to pursue an alternate route to the calibration of C_{cryo}: by using the inherent tunability of the calculable capacitor, we can directly compare C_{cryo} to the calculable capacitor C_{calc}. The second major achievement reported herein is a demonstration of such a comparison, with a relative uncertainty of about 4×10^{-8}.

II. Construction and Stability Results of a New 10-pF Capacitor

A. Construction

The design of the capacitor used herein is quite similar in philosophy to our previous 1-pF work [3]; the major change is from a parallel-plate to a coaxial design. As shown in Fig. 1, the cryogenic capacitor consists of three main parts: the ground shell, the high electrode, and the low electrode. The advantage of using the coaxial designs is that it reduces mechanical instabilities in the value of C_{cryo}, since to first order the value is independent of off-axis motion.

As indicated in the figure, this version has most of the same features as in our previous work: 1) vacuum-gap capacitor with
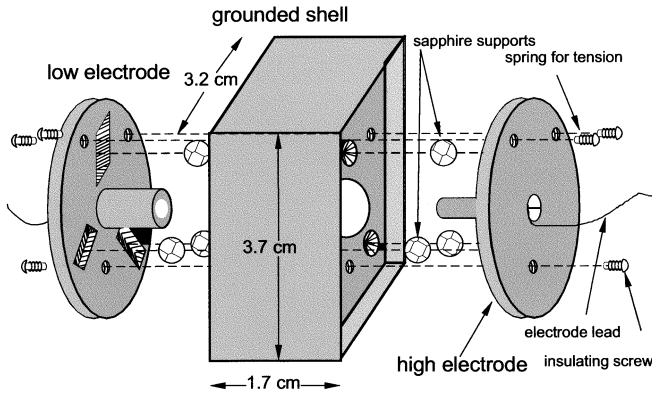
Fig. 1. Assembly diagram of a coaxial-plate capacitor, showing both electrodes as well as the grounded shell. The shell allows use in a three-terminal measurement configuration. Sapphire supports are used to maximize thermal conduction, while minimizing electrical leakage. Note that the two electrodes have no direct mechanical linkage; this ensures that there is minimal leakage directly between the two (this is the most detrimental leakage).
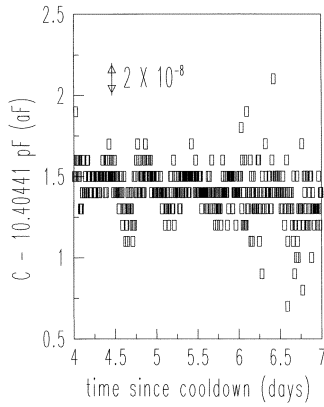

Fig. 2. Measurement of time dependence of the 10.3-pF vacuum-gap capacitor at 4.2 K; note that the short-term stability is excellent, with a drift less than 10^{-9}/hour.

Fig. 3. Schematic of the bridge circuit for comparison of C_{calc} to (a) the usual fused-silica transfer standard and to (b) the cryogenic capacitor. Because the cryogenic capacitor C_{cryo} does not have a nominal value, we use the inherent tunability of the calculable capacitor to achieve balance. As indicated in (b), we use a different tap of the ratio transformer and two different positions of the calculable capacitor, allowing comparison to a capacitor of arbitrary value. Thus, we can measure the value of the cryogenic capacitor in terms of F.

TABLE I
RESULTS OF REPEATED MEASUREMENTS OF C_{cryo}

trial	C (pF)
1	10.35312854
2	10.35312856
3	10.35312861
4	10.35312873
5	10.35312873
6	10.35312869

oxygen-free high conductivity (OFHC) Cu high and low electrodes separated by a grounded shell (achieves three-terminal capacitor configuration); 2) both electrodes mechanically supported by sapphire balls (to maximize thermal contact to shell) and held by nylon insulating screws; and 3) electrodes are not directly mechanically linked to each other (to minimize direct leakage). Instead, each electrode is mechanically supported by the ground shell; leakage from each electrode to the ground shell is much less detrimental than direct leakage from one electrode to the other.

B. Stability Results

The major extension in terms of measurement that we report herein is the direct comparison of C_{cryo} to C_{calc}. This direct comparison puts a tighter constraint on the time stability of the capacitor; in particular, we require a short-term relative drift smaller than 10^{-9}/hour, which is a factor of ten smaller than previously required.

In Fig. 2, we show a typical measurement of the capacitance as a function of time, done simply by monitoring the measurement of a commercial capacitance bridge, the
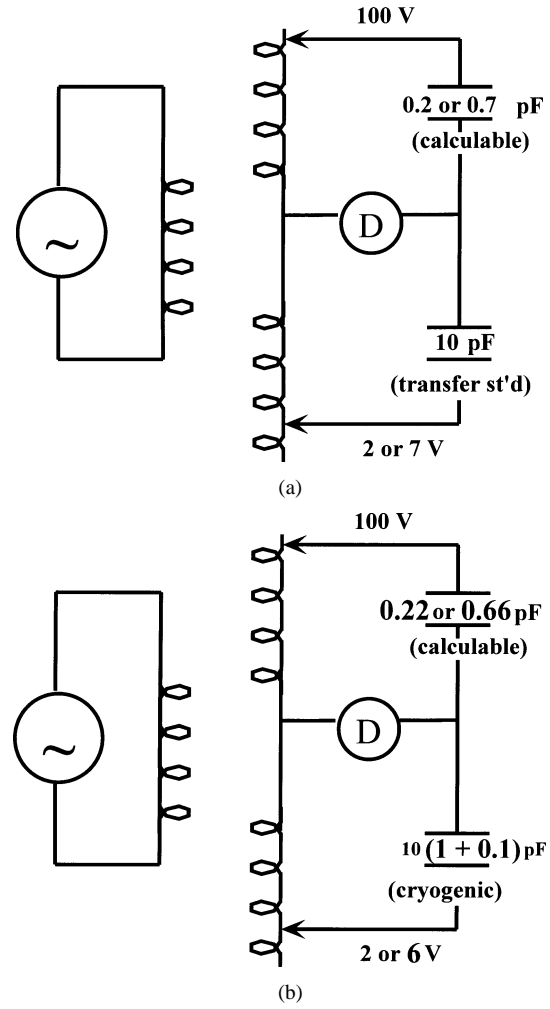
Andeen-Hagerling 2500A.[1] We note that, in order to obtain the excellent results (very low drift) displayed, it was necessary to thermally cycle the capacitors several times to either 77 K (liquid nitrogen) or 4 K (liquid helium). For the first few cycles, a capacitor typically showed large, nonrepeatable changes both with temperature and time at fixed temperature for a single

TABLE II
UNCERTAINTIES IN MEASUREMENT OF BOTH THE (LEFT) USUAL TRANSFER STANDARD [7] AND THE (RIGHT) CRYOGENIC CAPACITOR.
WE NOTE THAT, WITH ONLY MODEST ATTEMPTS TO MINIMIZE UNCERTAINTIES, WE HAVE REDUCED THE UNCERTAINTY
TO ONLY SLIGHTLY LARGER THAN THAT FOR THE USUAL MEASUREMENT

Source of uncertainty	Relative std. unc. for 112	Relative std. unc. for Cryogenic Capacitor
Variability of repeated observations	2×10^{-9}	5×10^{-9}
Geometrical imperfections in the calculable capacitor	15×10^{-9}	15×10^{-9}
Laser/Interferometer alignment	3×10^{-9}	3×10^{-9}
Frequency (loading) corrections	4×10^{-9}	30×10^{-9}
Microphonic coupling	5×10^{-9}	5×10^{-9}
Voltage dependence	5×10^{-9}	10×10^{-9}
Drift between calibrations/ failure to close	6×10^{-9}	6×10^{-9}
Transformer ratio measurement	2×10^{-9}	20×10^{-9}
Bridge linearity and phase adjustment	3×10^{-9}	3×10^{-9}
Detector uncertainties	2×10^{-9}	2×10^{-9}
Coaxial choke effectiveness	1×10^{-9}	1×10^{-9}
Temperature corrections for 10 pF capacitors	2×10^{-9}	0
Relative standard uncertainty	19×10^{-9}	42×10^{-9}

cycle, and also between cycles. After the first few cycles, the capacitance showed no time dependence at a fixed temperature, and a repeatable temperature dependence. It is likely that the nonrepeatability arises from anelastic or plastic deformation due to differential thermal contractions between the metal plates and the insulating elements; after the first few cycles, these deformations have relaxed, and all that is left is elastic shape changes which will not be hysteretic.

III. DIRECT COMPARISON OF CRYOGENIC AND CALCULABLE CAPACITORS

A. Basic Results

In this section, we discuss the results of a direct comparison of C_{cryo} to C_{calc}. As discussed in the Introduction, the motivation for this is to provide a link between the ECCS and the SI unit. Because C_{cryo} can drift relatively large amounts over long times, especially if it is thermally cycled, it will be necessary to run the pumping and the comparison phases of the ECCS (i.e., to pump electrons on to C_{cryo} and to compare C_{cryo} to C_{calc}) in quick succession. Since we do not yet have the capability to do both phases simultaneously, we have chosen to demonstrate the direct comparison of C_{cryo} to C_{calc} separately. The present results were obtained with C_{cryo} in a "dipper" cryostat inserted into liquid helium in a storage dewar. The capacitor sat in vacuum (about 10^{-7} torr).

The measurement circuit and configuration is schematically indicated in Fig. 3. Fig. 3(a) shows the standard configuration of the measurement bridge between the calculable capacitor and a transfer standard with a nominal value (typically, $10 \times (1 \pm 10^{-4})$ pF). Here, the two taps of the ratio transformer were chosen and have been calibrated to provide accurate comparison

using two standard positions of the calculable capacitor guard electrode [6].

However, the continuous positioning of the guard electrode means that the calculable capacitor has an inherent tunability; Fig. 3(b) shows how we make use of this. We note that the ratio transformer has taps so that, if the upper side is at 100 V, the lower can take on values of 2, 3, 4, 5, 6, or 7 V. To take advantage of the tunability, we use a different tap for one side of the ratio transformer (6 V instead of 7 V) and two different positions of the guard electrode (0.22 pF and 0.66 pF rather than 0.2 pF and 0.7 pF). Using this prescription, we can achieve a balance between the calculable capacitor and a cryogenic capacitor of arbitrary value between 10 and 11.6 pF.

We have achieved such a balance and thus demonstrated a comparison between C_{cryo} and C_{calc}. The type-A uncertainty is comparable to (perhaps slightly larger than) our usual comparison to a fused-silica 10-pF standard. This indicates that use of a vacuum-gap capacitor at cryogenic temperatures, and the associated cables (length 1.5 m) running over a temperature range from 300 to 4 K and back to 300 K, does not substantially degrade the bridge measurement.

One slight complication stems from the fact that, at present, we do not have the ability to measure the displacement of the guard electrode at all positions. In particular, our interferometer allows measurement over many optical wavelengths with an uncertainty of less than 1 nm, but we cannot count the number of wavelengths (fringes) between the two positions; this means that we cannot measure the absolute displacement for motions greater than 0.3 μm. However, since we know the fraction of a wavelength from the interferometer, if we can measure the relative value of C_{cryo} using the A-H 2500A within about 0.8×10^{-6} (corresponding to the change in C_{calc} for 1/2 fringe) immediately before and after the bridge comparison, we can

combine the two measurements to derive the value. We have done this for the measurements reported herein.

We note that this procedure requires pushing the capabilities of the A-H 2500A beyond its rated uncertainty, which we have accomplished as follows. Although the A-H 2500A has an absolute uncertainty of 3×10^{-6}, its local nonlinearity is much better than this. In particular, we have found (not published) that, if we calibrate the A-H 2500A by measuring a known capacitor at 10 pF and then measure an unknown capacitor with value between 10 pF and 11 pF, we can derive the correct value of the unknown by applying a correction from the calibration. We have found that the relative error in following this prescription is less than 0.2×10^{-6}, and thus we can easily measure the value of C_{cryo} using the A-H 2500A within 0.8×10^{-6}.

Table I shows the results of repeated measurements following this prescription; the measurements encompassed about three hours.

B. Uncertainty

Table II lists the type-A and -B uncertainties for our measurement, as well as those for our standard measurement of the 10-pF fused-silica standard [7]. We note that, for this demonstration, we made only a modest attempt to optimize the uncertainties; we expect that ultimately the uncertainties should be equal for both measurements.

IV. CONCLUSION

We have proposed and demonstrated a new cryogenic capacitor with much larger value (10 pF). This should allow a more useful functioning of the "turnkey" system based on the ECCS, as well as a more accurate calibration of the cryogenic capacitor. The capacitor has quite good short-term stability, and we have measured the value in the SI with a relative uncertainty of about 4×10^{-8}. We plan on combining this direct comparison with the pumping phase of the ECCS; one payoff will be to close the metrology triangle.

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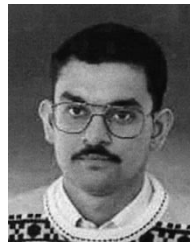
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